Vibration Energy Harvesting Using Piezoelectricity: Single Segment, Multi-Segment, and Circuit Modeling

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What is Energy Harvesting?

- Many devices (sensors, transmitters) exist in environments with untapped sources of energy
- Without increasing the size/weight of device or impacting environment, energy can be tapped for intermittent use
A Brief History of Piezoelectric Energy Harvesting

- Impetus: battery technology has been outpaced by the devices that require wireless energy
- Often energy exists in the environment of devices that can be harvested without significant impact to the source of the energy
- Energy harvesting is a key enable of extremely long-term, self-reliant wireless systems
- Specifically, piezoelectric material allows the harvesting of energy from strain

Power Harvesting Application: Insect Cyborg Sentinels

New contributions:
- Implantation of microsystems and active components at the pupa developmental stage.
- On-board directional control of insect motion through behavioral and/or direct stimulation
- Systems integration of life with power harvesting, navigation, and sensor suites.

Experimental flight data for motion of moth:
- 3 mm peak-to-peak amplitude
- 25 Hz frequency
- 1.2-2.2g RMS acceleration (based on payload size), 5.2g max acceleration
- 3.4m/s confined, 5.3m/s open-space
- ~ 100mW (RMS) mechanical power output

Constraints on power harvesting system:
- Limited size: < 5cm
- Limited weight: < 1g
- Limited available base excitation: 1.2-2.2g RMS


Power Harvesting Application: Bio-motions

Design Challenges
- Batteries do not offer sufficient energy density to power hand-held devices for long term missions (multi-month/multi-year)
- Vibration sources are often time-varying, non-stationary, and intermittent
- Motion from humans and animals is often low frequency (0.25-20 Hz), so traditional, resonant devices would be too large or heavy

Outline

1. The piezoelectric effect

2. Modeling a single Euler-Bernoulli beam

3. Modeling multi-beam structures

4. Introducing nonlinearities for increased bandwidth

5. Comparison of active and passive circuits
The Piezoelectric Effect

- Crystal lattice structure has inherent electric dipole density which deforms as lattice deforms
- Change in dipole density strength and/or direction causes an apparent charge separation
- When connected to a circuit, the material functions as a time-varying capacitor. Charge separation can be used to produce a current or voltage.
- Stress and strain is coupled bi-directionally to voltage and current:

\[
\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix}
+
\begin{bmatrix}
0 & 0 & d_{31} \\
0 & 0 & d_{32} \\
0 & 0 & d_{33}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}

= \begin{bmatrix}
\Delta E_1 \\
\Delta E_2 \\
\Delta E_3
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & d_{11} & 0 & 0 \\
0 & 0 & d_{22} & d_{23} & 0 & 0 \\
d_{31} & d_{32} & d_{33} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta L_1 \\
\Delta L_2 \\
\Delta L_3
\end{bmatrix}
\]

“Piezoelectricity,” en.wikipedia.org/wiki/Piezoelectricity [retrieved 01/09/12]

Axial deformation causes change in voltage and vice versa.

Electric dipoles in crystal structure cause an apparent charge on surfaces, much like parallel plate capacitors.
The Piezoelectric Effect, planar case

\[
T_1 = c_{11}^E S_1 - e_{31} E_3
\]

\[
D_3 = e_{31} S_1 + \varepsilon_{33}^S E_3
\]

- \( T_1 \) = stress
- \( D_3 \) = electrical displacement
- \( S_1 \) = strain
- \( E_3 \) = electric field strength
- \( c_{11} \) = Young’s Modulus (constant \( E \))
- \( e_{31} \) = piezoelectric coupling coefficient
- \( \varepsilon_{33} \) = dielectric constant (constant \( S \))

The behavior of piezoelectric materials is a combination of standard mechanical response (stress-strain), capacitive electrical response, and electromechanical coupling.
Piezoelectric Energy Harvester Geometry

- Geometry chosen is a cantilevered beam with piezoelectric patches (or layers) bonded to the upper and lower surfaces
- Simple: only one mounting point
- Low natural frequencies (compared to other boundary conditions)
- Easily tunable with a tip mass
- Fundamental (i.e. lowest natural frequency) mode has no strain nodes

Parallel bimorph configuration is employed in order to increase charge displaced by the beam bending


“Capacitor,” en.wikipedia.org/wiki/Capacitor [retrieved 01/09/12]
Euler-Bernoulli Beam Modeling

Assumptions:
- Within each layer, material is linear and homogeneous
- Plane sections remain plane (small shear deformations)
- Beam is vibrating in its plane of symmetry (no twisting)
- Rotary inertia and shear deformations are small

These assumptions require the length of the beam to be at least an order of magnitude greater than the other two dimensions.

Balance of forces:
\[
\frac{\partial V(x,t)}{\partial x} + f(x,t) = (\rho A) \frac{\partial^2 w(x,t)}{\partial t^2}
\]

Balance of moments:
\[
\frac{\partial M(x,t)}{\partial x} = -V(x,t)
\]

\[\Rightarrow (\rho A) \frac{\partial^2 w(x,t)}{\partial t^2} + \frac{\partial^2 M(x,t)}{\partial x^2} = f(x,t)\]
Euler-Bernoulli Beam Modeling (cont.)

\[ M(x, t) = T_1 = c_{11}S_1 - e_{31}E_3 \]

\[ T_1 = cS_1 \]

\[ T_1 = c_{11}S_1 - e_{31}E_3 \]

\[ M(x, t) = \int^{t_s/2}_{-t_s/2-t_p} T_1bydy + \int^{t_s/2}_{-t_s/2} T_1bydy + \int^{t_s/2+t_p}_{t_s/2} T_1bydy \]

\[ = -\left[ \int^{t_s/2}_{-t_s/2-t_p} c_{11}by^2dy + \int^{t_s/2}_{-t_s/2} c_{11,s}by^2dy + \int^{t_s/2+t_p}_{t_s/2} c_{11}Eby^2dy \right] \frac{\partial^2 w(x, t)}{\partial x^2} \]

\[ - \left[ \int^{t_s/2}_{-t_s/2-t_p} e_{31}bydy - \int^{t_s/2+t_p}_{t_s/2} e_{31}bydy \right] \nu(t) [H(x - x_L) - H(x - x_R)] \]

\[ = \left\{ c_{11,s}b \frac{t_s^3}{12} + 2c_{11}b \left[ \frac{t_p^3}{12} + t_p \left( \frac{t_p + t_s}{2} \right)^2 \right] \right\} \frac{\partial^2 w(x, t)}{\partial x^2} + -e_{31}b(t_s + t_p) \nu(t) [H(x - x_L) - H(x - x_R)] \]

Effective bending stiffness

Piezoelectric coupling coefficient

\[ \rho \]

piezoelectric layer

substrate layer

piezoelectric layer
Modal Analysis of Piezoelectric Bimorph

- Assume that the beam motion can be written as a summation of orthogonal modes:

\[ w(x, t) = \sum_{i=1}^{\infty} \phi_i(x) r_i(t) \]
Modal Analysis (cont.)

- Then a separate ODE can be written for each mode:

\[
\frac{d^2 r_k(t)}{dt^2} + 2\zeta_k \omega_{SC,k} \frac{dr_k(t)}{dt} + \omega_{SC,k}^2 r_k(t) + \Theta_k v(t) = -\rho A \gamma_k \frac{d^2 y(t)}{dt^2}, \forall k \in \mathbb{N}
\]

\( \zeta_k \) = modal damping

\( \omega_{SC,k} \) = modal short - circuit natural frequency \((v(t) = 0)\)

\( \gamma_k \) = base motion modal participation factor = \(\int_0^L \phi_k(x)dx\)

\( \Theta_k \) = modal coupling coefficient = \(\sum_{j=1}^{n_p} e_{31} hb_{p,j} \left( \frac{d\phi_k(x_{R,j})}{dx} - \frac{d\phi_k(x_{L,j})}{dx} \right)\)

- There is an additional equation relating the electrical states to the modes:

\[
q(t) = \sum_{i=1}^{\infty} \Theta_i r_i(t) - C_0 v(t), \text{ where } C_0 = \text{piezo static capacitance } = \frac{\varepsilon_{33}}{t_p} \sum_{j=1}^{n_p} b_{p,j} (x_{R,j} - x_{L,j})
\]
Frequency Response Functions

Tip deflection $w_{rel}/Y$

Voltage $V/(Y \omega^{1.5})$

Current $I/(Y \omega^{1.5})$

Power $P/(Y^{2.3} \omega)$

- $R = 0 \Omega$
- $R = 1000 \Omega$
- $R = 5000 \Omega$
- $R = 100000 \Omega$
- $R = \infty \Omega$

![Circuit Diagram]

$\omega/\omega_1$
A Multitude of Structural Designs for Energy Harvesting

The Goal: A Single Solution Procedure for all Designs

Freedoms

- Each segment can have different material properties, dimensions, and layout (e.g. unimorph, bimorph, etc.)
  \[ (\rho A) = \text{mass/length} \quad (EI) = \text{transverse bending stiffness} \]
  \[ \vartheta = \text{electromechanical coupling factor} \quad C = \text{intrinsic capacitance} \]

- Each joint can have a different mass, moment of inertia, and angle
  \[ m = \text{lumped mass} \quad I = \text{lumped moment of inertia} \quad \theta = \text{joint angle} \]

- External loads can be base excitation, point loads, or distributed loads
- Multiple host connections can be prescribed
The Goal: A Single Solution Procedure for all Designs

Methodology

- Transverse deflections \( w(x,t) \) and axial deflections \( u(x,t) \) are decoupled:

\[
w(x,t) = \sum_{r=1}^{\infty} \eta_r(t)\phi_r(x), \quad u(x,t) = \sum_{r=1}^{\infty} \eta_r(t)\psi_r(x)
\]

- A state vector is prescribed at each position along the length of the structure:

\[
z = \begin{bmatrix} \psi & N & \phi & \frac{d\phi}{dx} & M & V \end{bmatrix}^T
\]

- A linear system of ODEs is derived:

\[
\frac{dz}{dx} = A(x)z(x)
\]

- State transition matrices are calculated:

\[
z(x_2) = \Phi(x_2, x_1)z(x_1)
\]

- Example state transition matrices:

\[
z(x_2) = F_1(x_2 - x_1)z(x_1)
\]

\[
z(x_4) = F_3(x_4 - L_2)P_2F_2(L_2 - x_3)z(x_3)
\]
Our Goal: A Single Solution Procedure for all Designs

Assumptions

- Each segment has uniform properties (continuously varying properties can be approximated with a large number of uniform segments or approximate mode shapes)
- No out-of-plane motions/torsional coupling (this can be added by adding more states)
- No axial deformations (this can be added by adding more states)
- Each segment is modeled using Euler-Bernoulli beam assumptions (this can be relaxed by adding more states)
Starting Point: A Single Euler-Bernoulli Beam Segment

\[
(\rho A) \frac{\partial^2 w(x,t)}{\partial t^2} + (EI) \frac{\partial^4 w(x,t)}{\partial x^4} + g \left[ \frac{d\delta(x-L_L)}{dx} - \frac{d\delta(x-L_R)}{dx} \right] v(t) = f(x,t) \quad (1)
\]

\[
q(t) = g \left[ \frac{\partial w(x,t)}{\partial x} \bigg|_{x=L_R} - \frac{\partial w(x,t)}{\partial x} \bigg|_{x=L_L} \right] - Cv(t) \quad (2)
\]

\[
(\rho A) \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial N(x,t)}{\partial x} = 0 \quad (3)
\]

<table>
<thead>
<tr>
<th></th>
<th>Unimorph</th>
<th>Bimorph</th>
<th>Bare Substructure</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\rho A))</td>
<td>(b_l(\rho t_s + \rho t_p))</td>
<td>(b_l(\rho t_s + 2\rho t_p))</td>
<td>(b, \rho, t_s)</td>
</tr>
<tr>
<td>((EI))</td>
<td>(\frac{b_s c^2 t_s^4 + 4c_s^2 c_t t_s^2 t_p^4 + 64c_s^2 c_t t_s^2 t_p^4 + 256c_s^2 c_t t_s^2 t_p^4}{12 c_s^2 t_s + c_t} t_p^2)</td>
<td>(c_s b_t \left( t_s^4 + t_p^4 \left( \frac{t_s + t_p}{2} \right)^2 \right) )</td>
<td>(c_s b_t t_p^2)</td>
</tr>
<tr>
<td>(g)</td>
<td>(-\frac{e_s b_c c_t(t_p + t_s)}{2 c_s^2 t_s + c_t}t_p)</td>
<td>(-e_s b_c(t_p + t_s))</td>
<td>0</td>
</tr>
<tr>
<td>(C)</td>
<td>(\frac{e_{s1} b_L}{t_p})</td>
<td>(\frac{2e_{s1} b_L}{t_p})</td>
<td>0</td>
</tr>
</tbody>
</table>
Field Transfer Matrix: Relating States Along Beam Segments

- To compute the field transfer matrix, external loads, including electrical, are dropped

\[
\frac{\partial V(x,t)}{\partial x} = (\rho A)_j \frac{\partial^2 w(x,t)}{\partial t^2}
\]
\[
\frac{dM(x,t)}{dx} = -V(x,t)
\]

\[
(\rho A)_j \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial N(x,t)}{\partial x} = 0
\]
\[
M(x,t) = (EI)_j \frac{\partial^2 w(x,t)}{\partial x^2}
\]

- These equations can be combined into the following linear system:

\[
\begin{bmatrix}
\psi \\
N \\
\phi \\
\frac{d\phi}{dx} \\
M \\
V
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & -\rho A'_j \omega^2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\psi \\
N \\
\phi \\
\frac{d\phi}{dx} \\
M \\
V
\end{bmatrix}
\]
Field Transfer Matrix (cont.)

- The field transfer matrix can then be computed in closed form using the matrix exponential:

\[
F_j(\Delta x) = e^{A_j \Delta x}
\]

\[
F_j(\Delta x) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
\Delta x(\rho A)_j \omega^2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & c_0 & \Delta x c_1 & \frac{(\Delta x)^2}{(EI)_j} c_2 & -\frac{(\Delta x)^3}{(EI)_j} c_3 \\
0 & 0 & \frac{(\Delta x)^3(\rho A)_j \omega}{(EI)_j} c_3 & c_0 & \frac{\Delta x}{(EI)_j} c_1 & -\frac{(\Delta x)^2}{(EI)_j} c_2 \\
0 & 0 & (\Delta x)^2(\rho A)_j \omega^2 c_2 & (\Delta x)^3(\rho A)_j \omega^2 c_3 & c_0 & -\Delta x c_1 \\
0 & 0 & -\Delta x(\rho A)_j \omega^2 c_1 & -(\Delta x)^2(\rho A)_j \omega^2 c_2 & -\frac{(\Delta x)^3(\rho A)_j \omega^2}{(EI)_j} c_3 & c_0
\end{bmatrix}
\]
Point Transfer Matrix: Relating State Across a Discontinuity

For non-zero joint angles, the transverse and axial dynamics become coupled
Eigensolution

- Product of all field and point transfer matrices gives the system transfer matrix, which relates the states at the base to the states at the tip

\[
\begin{bmatrix}
\psi(L_n) \\
N(L_n) \\
\phi(L_n) \\
d\phi(L_n)/dx \\
M(L_n) \\
V(L_n)
\end{bmatrix} = \prod_{j=1}^{n} \left[P_{n-j}F_{n-j+1}(L_{n-j+1} - L_{n-j})\right] \begin{bmatrix}
\psi(0) \\
N(0) \\
\phi(0) \\
d\phi(0)/dx \\
M(0) \\
V(0)
\end{bmatrix}
\]

- Fixed BC applied at base, free BC applied at tip
- Eigenvalues of \( U \) give the natural frequencies of the structure
Case Study #1: Partial Piezo Layer Coverage

(a) [Image of host structure, piezoelectric layers, and inactive substructure]

(b) [Image of host structure, inactive substructure, and piezoelectric layers]

Graphs showing the relationship between $x_{div}$ (m) and $\omega_1$ (rad/s) for different $t_p$ values: $t_p = 0.05 t_s$, $t_p = 0.1 t_s$, $t_p = 0.25 t_s$, $t_p = 0.5 t_s$, $t_p = 1 t_s$, $t_p = 2 t_s$, and $t_p = 3 t_s$.

Graphs showing the relationship between $x_{div}$ (m) and $\psi_1$ (rad/s) for different $t_p$ values: $t_p = 0.05 t_s$, $t_p = 0.1 t_s$, $t_p = 0.25 t_s$, $t_p = 0.5 t_s$, $t_p = 1 t_s$, $t_p = 2 t_s$, and $t_p = 3 t_s$. 
Case Study #1: Partial Piezo Layer Coverage

(a)

(b)
COMSOL FEA Validation

COMSOL finite-element model of partial piezoelectric coverage beam

Average FEA domain element statistics

<table>
<thead>
<tr>
<th>no. of elements</th>
<th>254</th>
</tr>
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<tbody>
<tr>
<td>minimum element quality</td>
<td>0.701</td>
</tr>
<tr>
<td>average element quality</td>
<td>0.9049</td>
</tr>
<tr>
<td>element area ratio</td>
<td>0.007213</td>
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<tr>
<td>mesh area</td>
<td>$1.75 \times 10^{-4}$ m$^2$</td>
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</tbody>
</table>

First three mode shapes, shaded by x-strain

<table>
<thead>
<tr>
<th>$x_{div}/L$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode</td>
<td>TMM</td>
<td>FEA</td>
<td>rel. error</td>
<td>TMM</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>43.07</td>
<td>42.97</td>
<td>0.23%</td>
<td>72.50</td>
<td>72.08</td>
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<tr>
<td>2</td>
<td>269.94</td>
<td>269.85</td>
<td>0.03%</td>
<td>426.01</td>
<td>424.20</td>
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<tr>
<td>3</td>
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<td>1051.15</td>
<td>1049.14</td>
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<tr>
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<td>1759.02</td>
<td>1751.25</td>
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<tr>
<td>5</td>
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<td>0.18%</td>
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<td>2843.51</td>
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<td>6</td>
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<td>3648.09</td>
<td>0.26%</td>
<td>4447.26</td>
<td>4408.76</td>
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</table>
Case Study #2: Variable Center Joint Angle

The diagrams illustrate the relationship between the angular position $\theta_1$ and the angular velocity $\omega_1$ for different values of $t_p = \{0.05 t_s, 0.1 t_s, 0.25 t_s, 0.5 t_s, 1 t_s, 2 t_s, 3 t_s\}$.
## COMSOL FEA Validation

**COMSOL finite-element model of L-shaped piezoelectric bracket**

First three mode shapes, shaded by x-strain

<table>
<thead>
<tr>
<th>mode</th>
<th>TMM</th>
<th>FEA</th>
<th>rel. error</th>
<th>TMM</th>
<th>FEA</th>
<th>rel. error</th>
<th>TMM</th>
<th>FEA</th>
<th>rel. error</th>
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<td>88.05</td>
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<td>111.13</td>
<td>109.60</td>
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<td>165.42</td>
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<td>233.94</td>
<td>230.63</td>
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<td>410.51</td>
<td>1.50%</td>
<td>302.62</td>
<td>298.22</td>
<td>1.48%</td>
<td>249.15</td>
<td>245.57</td>
<td>1.46%</td>
<td>234.01</td>
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<td>1439.68</td>
<td>1.67%</td>
<td>1494.14</td>
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<td>2369.15</td>
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<td>4743.14</td>
<td>4639.48</td>
<td>2.23%</td>
<td>4778.84</td>
<td>4660.78</td>
<td>2.53%</td>
<td>4958.97</td>
<td>4778.31</td>
<td>3.78%</td>
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<td>5938.58</td>
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<td>3.35%</td>
<td>5866.12</td>
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<td>2.65%</td>
<td>5850.11</td>
<td>5715.54</td>
<td>2.35%</td>
</tr>
</tbody>
</table>
Frequency Up-Conversion Techniques

- Also termed “two-stage” or “mechanical/magnetic rectification”
- Primary structure is induced to move or vibration by external stimuli
- Motion periodically induces (or “plucks”) a secondary structure to vibrate at its fundamental frequency

Rotary

Linear

MEMS


Effect of Magnet Strength

- Force magnitude is normalized by the static tip load required to induce a 0.1% maximum strain in the beam

\[ F_{\text{max}} = \frac{(EI)_{\text{eff}}}{\left(\frac{t_s + t_p}{2}\right)L} (0.001) = 7.03 \text{ N} \]

\[ F_{\text{mag}} = 0 \quad F_{\text{mag}} = 0.5F_{\text{max}} \]

Effect of Ferrous Structure Spacing

- Ferrous structure spacing does not have as pronounced or consistent effect on power harvested
- There are two conflicting effects: closer spacing means more impulses, but a greater chance of “killing” the oscillations

$d_m = 5\text{mm}$  \hspace{1cm}  $d_m = 10\text{mm}$
One-Mode Model Coupling with Electrical Circuit

\[ M \ddot{u} + \eta_m \dot{u} + Ku + \Theta V_p = F(t) \]
\[ -\Theta \dot{u} + C_0 V_p = -I(t) \]
\[ I_p = \Theta \dot{u} = \Theta u_0 \omega \sin(\omega t) = \hat{I}_p \sin(\omega t) \]

- Note that coefficients in the displacement equation are all “effective”
- They are functions of beam properties, piezo properties, beam geometry/layout, boundary conditions, location of position coordinate \( u \), etc.
- Often, if coupling is “small”, transducer is considered an ideal current source in parallel to capacitor

Circuit Topology #1: Direct Charging of a Storage Capacitor

A recursive relationship for $V_C$ can be found:

$$V_{C,j+1} = V_{C,j} \left(1 - \frac{2C_0}{C_u + C_0}\right) + \frac{2C_0}{C_u + C_0} \tilde{u}_{0,j}.$$  

$$\tilde{u}_{0,j} = \frac{1}{2} - \frac{2\Theta^2}{\eta_mC_0\omega\pi} \left(\frac{C_u}{C_u + 2C_0}\right) \tilde{V}_{C,j} + \frac{4\Theta^2}{\eta_mC_0\omega\pi} \left(\frac{C_u}{C_u + 2C_0}\right) \tilde{V}_{C,j} - 1.$$

The constant amplitude case can be written as a convergent geometric series:

$$V_{C,i} = \sum_{n=0}^{i-1} A^n B = \frac{B(A^i - 1)}{A - 1}, \text{ for } i \geq 1,$$

where

$$A = 1 - \frac{2C_0}{C_u + C_0} = \frac{C_u - C_0}{C_u + C_0}; \quad B = \frac{2\hat{I}_p}{\omega(C_u + C_0)}.$$

$$V_{C,\infty} = \lim_{i \to 2} \left(\sum_{n=0}^{i-1} A^n B\right) = \frac{B}{1 - A} = \frac{\hat{I}_p}{\omega C_0} = \tilde{V}_{OC}.$$

The constant excitation case also converges:

$$\tilde{V}_{C,\infty} = \tilde{V}_{C,\infty} \left(1 - \frac{2C_0}{C_u + C_0}\right) + \frac{2C_0}{C_u + C_0} \tilde{u}_{0,\infty} \quad \tilde{V}_{C,\infty} = \tilde{V}_{OC}.$$
Circuit Topology #2: Synchronized Switching and Discharging to a storage Capacitor through an Inductor (SSDCI)

Waveforms of the SSDCI case with $C_u=10C_0$


Experimental Validation (weak coupling)

Storage capacitor voltage, \( \frac{C_u}{C_0} = 779.349 \)

![Graph showing capacitor charging voltage versus time for all three cases](image)

### Capacitor charging voltage versus time for all three cases

- SSDC case had highest losses through rectifier and switch
- SSDCI case charges more than twice as fast as the other two cases
- SSDCI case maintains a flatter power transfer due to partial isolation of input (piezo) and output (storage capacitor) stages
- Constant amplitude assumption predicts power harvesting dynamics well for weakly coupled beams

Charging Power vs. Voltage, \( \frac{C_u}{C_0} = 779.349 \)

![Graph showing power charging the storage capacitor versus voltage for all three cases](image)
Experimental Validation (strong coupling)

- Constant amplitude assumption is not valid for more strongly coupled beams
- Amplitude decreases the most significantly during the peak charging power, confirming the conservation of energy assumption
- Symmetry is lost at higher coupling because current input is no longer constant

Conclusions

• Complex systems can often be reduced in order (# of states) if you can show that a state is very small compared to the others or very fast/slow
• But you need to check your assumptions (e.g. single-mode excitation, plane strain, low electromechanical coupling, etc.)
• Power transduction efficiency depends not only on material but on geometry and placement
• EH from a single frequency is relatively easy, multiple frequencies or broadband excitation is quite hard
• An active circuit can extract up to ~10x the power as a passive circuit, but its power consumption could outweigh its benefits
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