Non-Linear Dynamics of Coupled MEMS Oscillators

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Abstract:

An optically thin resonator suspended over a substrate and illuminated with a laser forms an interference field that couples deflection of the resonator to absorption within it. For high enough laser power, such resonators have been shown to self-oscillate due to the feedback between heating and deflection. Our research focuses on understanding the driving forces of self-oscillation using theoretical models, finite element method (FEM) analysis, and experimental observation. Theoretical models have suggested the possibility of previously unreported high amplitude self-oscillation modes. FEM analysis has indicated the importance of pre-stress in driving self-oscillation. Experimental results suggest the presence of thermal noise driven vibrations before the onset of limit cycle oscillations.

Summary of Research:

Thermally driven MEMS limit cycle oscillators depend on a feedback between heating and displacement. The mechanism coupling centerline heating to displacement is well understood [1], however the process by which heating causes displacement is not. Previous modeling work has indicated that thermo-mechanical coupling drives limit cycle oscillations [2], but did not examine the nature of the coupling.

In order to understand this coupling, we studied the deflection of doubly-supported micro-beams under steady state heating using an FEM model. Results show that deflection due to heating in doubly supported beams comes from compressive stresses at the beam support due to thermal gradients across the length of the device (see Figure 1). Asymmetry of the beam support causes a phenomenon known as “imperfection buckling” where stress due to thermal expansion causes a deflection away from the substrate. This deflection is initially small but grows rapidly at the buckling load (see Figure 2). As a result, barely post buckled beams should have the strongest thermo-mechanical coupling, leading to the lowest threshold power for self-oscillation.

Previous work on cantilevered beams [3] suggests that for devices which do not support transverse compression, deflection due to heating is directed down towards the substrate and caused by thermal gradients across the thickness of the device at the beam support. This difference in displacement direction should cause a difference in phase of feedback, the effect of which is unknown.

Examination of the coupled oscillator equations typically used to model thermally driven MEMS limit cycle oscillations predicts the possibility of multiple stable limit cycle oscillations. Previous work on the model had assumed

Figure 1: Results of steady state thermal and small deflection structural analyses for a 7 µm long doubly supported beam. Quarter-symmetry is used to reduce the computational burden. Note that the deflection is up, away from the substrate.
small displacement and expanded the function describing the interference field in a Taylor series expansion, keeping only the first few terms [2,4] and losing the periodicity in the process. In our work, we examine the nature of solutions to the full non-linear equations using numerical continuation, and show that a periodic interference field leads to multiple stable limit cycles for lightly damped resonators. In addition, for high laser powers, period doubling and chaos have been observed in the model. Devices exhibiting multiple stable limit cycle oscillations would allow tuning between distinct frequency bands, and within them, in applications such as GPS receivers. Due to the small gap between device and substrate in our samples (~ 0.75 λ), higher amplitude limit cycle oscillations could not be experimentally verified in our devices.

Finally, experimental results have verified a previous analytic result that the second mode oscillation of a clamped-clamped beam is softening. However, measurement of the amplitude of oscillation as a function of laser power for our devices suggests that the amplitude of oscillation grows continuously from zero after a critical laser power threshold is reached. This type of dependence of amplitude on laser power suggests that the Hopf bifurcation leading to limit cycle oscillation is supercritical, or thermal vibrations are present and being picked up, a result which contradicts past work [5,6] on different device geometries (paddles, disks). Results are being independently verified using an alternate test setup.

References:


